Over parameterisation and optimisation approaches for identification of nonlinear stochastic systems described by Hammerstein-Wiener models

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Abstract: This paper proposes two iterative procedures based on over-parameterisation and optimisation approaches for the identification of nonlinear systems which can be described by Hammerstein-Wiener stochastic models. In this case, the dynamic linear part of the considered system is described by ARMAX mathematical model. The static nonlinear block is approximated by polynomial functions. The first procedure is based on a combination of the prediction error method by using the recursive approximated maximum likelihood estimator (RAML), the singular value decomposition (SVD) approach and the fuzzy techniques in order to estimate the parameters of the considered process. As for the second procedure, it includes an appropriate representation named as generalised orthonormal basis filters (GOBF) in order to reduce the complexity of the considered system. The parametric estimation problem is formulated using the recursive extended least squares (RELS) algorithm incorporated with the singular value decomposition (SVD) and fuzzy techniques in order to segregate the coupled parameters and improve the estimation quality. The validity of the developed approaches is proved by considering a nonlinear hydraulic process simulation.

Keywords: nonlinear stochastic systems; Hammerstein-Wiener models; ARMAX model; GOBF representation; parametric estimation; prediction error method; SVD approach; fuzzy technique; hydraulic process simulation.

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1 Introduction

Several systems applications have nonlinear structures. In the literature, the description and the control of the nonlinear dynamic systems have enticed the attention of many researchers and engineers. Consequently, numerous representations of mathematical models, which can describe the dynamics of these processes, have been considered in several research studies (Pottmann and Pearson, 1998; Haber and Keviczky, 1999; Giri and Bai, 2010; Billings, 2013; Elloumi and Kamoun, 2016b; Bauchspiess et al., 2017; etc.). Among the representations being used, we can mention the block oriented structures, which have been paid much attention due to a specific construction. They are generally subdivided into a linear dynamic block and a static nonlinear one. Thereby, the main advantage of this portrayal is to depict the dynamic linear block by an appropriate mathematical representation allowing the adequate description of the considered system dynamics, like: the transfer function (Wigren, 1993), the orthonormal bases filter (Gomez and Baeyens, 2004), the Kautz functions (Tötterman and Toivonen, 2009) and the ARMAX mathematical model (Ma and Liu, 2016). Concerning the description of the static nonlinear part, we can quote the neural networks representation (Arto et al., 2001), polynomial functions (Kamoun, 2013; Elloumi and Kamoun, 2017), hysteresis-saturation (Salhi and Kamoun, 2017a), and so on.

In the published research writings, the block-oriented structure can be categorised into two major classes: the Hammerstein model that is composed of a static nonlinear block pursued by a dynamic linear one, and the Wiener structure which is represented by the inverse of the first configuration (Chaari et al., 2006; Salhi and Kamoun, 2017b; Liu et al., 2018). Based on these two structures, extended forms, which are named complex blocks-oriented models, have been developed in the literature, like the Wiener-Hammerstein and Hammerstein-Wiener structures. The first type is constituted of a nonlinear block situated between two linear blocks (Vörös, 2007; Falck et al., 2012; Han and Raymond, 2012). The second type is composed of a linear block included between two static nonlinear blocks.

Several researches dealing with the Hammerstein-Wiener structure have been developed by many researchers and automation engineers worldwide. It has been shown that it was successfully applied to describe a large number of nonlinear processes in multiple practical applications like: submarine detection (Abrahamsson et al., 2007), RF power amplifier modelling (Taranga et al., 2010), chemical processes (Giri and Bai, 2010), so forth. However, the parametric estimation has turned out to be tougher since the Hammerstein-Wiener model not only includes the unknown parameters of linear and nonlinear subsystems but also contains two unmeasurable intermediate variables. In the same context, there was a little contribution to relevant works that developed methods to modelling and estimating schemes concerning the Hammerstein-Wiener model such

as Bai (1998) who applies the two-stage algorithm in order to identify an approximate model of Hammerstein-Wiener. Based on this algorithm, several techniques have been suggested, for instance: blind approaches (Bai, 2002), iterative algorithm (Crama and Schoukens, 2004), extended stochastic gradient identification algorithms (Wang and Ding, 2008), hierarchical least squares estimation algorithm (Wang and Ding, 2012), extended Kalman filter (Mansouri et al., 2011), over parameterisation and iterative approaches (Schoukens et al., 2012), unscented Kalman filter (Mazaheri et al., 2014) and generalised extended stochastic gradient (GESG), a multi-innovation GESG, a data filtering-based GESG and MI-GESG algorithms (Wang and Ding, 2015). Nevertheless, the application of numerous approaches (Bai, 1998; Abrahamsson et al., 2007; Wang and Ding, 2008, 2012, 2015; Mansouri et al., 2011; Mazaheri et al., 2014) requires a particular representation related to a Hammerstein-Wiener block system but not exactly equal. Because this structure does take into account the signal contributes directly to the output signal, without being distorted by the output nonlinearity. Also, as can be seen, it is usual that these algorithms limited their analysis of Hammerstein-Wiener with unknown constant parameters, without being transformed in the generalised orthonormal basis filters (GOBF), while the most real systems posed unknown time-varying parameters. Let us note that in the engineering literature, a few results of the modelling and the estimation about the Hammerstein-Wiener stochastic systems have been developed and published. It is not surprising that only a few methods dealing with Hammerstein-Wiener system identification using stochastic approaches are available, as the maximum-likelihood method (Wills et al., 2013) and the extended least squares approach.

In relation to this, it is important to emphasise that the work here is distinguished from those and other precedent contributions to compensate for the aforementioned shortcomings.

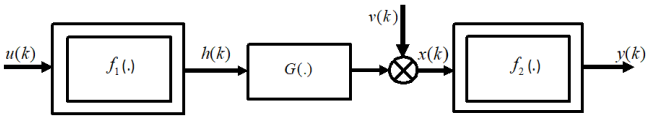
This paper introduces a more applicable representation of the Hammerstein-Wiener mathematical model that describes the dynamics of a nonlinear system operating in stochastic environment. The basic liabilities associated with the modelling and the identification of the Hammerstein-Wiener model reside not only in the attendance of the unmeasurable intermediate variables, which are acting between the linear and the nonlinear blocks, but also in the presence of the stochastic disturbance before the second nonlinearity and the considered structure which is manifested in the cascade connection of the three blocks. Under such circumstances, the main idea of this article is to put forward two approaches to identify and to simplify the formulation of the parametric estimation problem for the Hammerstein-Wiener mathematical model. Supplementary, the purpose of applying these strategies is to enhance the capability and flexibility of Hammerstein-Wiener models in predicting the nonlinear dynamics behaviour of the complicated systems with unknown time-varying parameters. Underpinning the central idea of these approaches is to describe the linear part and the correlated noise by different structures, on the one hand, and to represent the nonlinear parts by polynomial functions with a known order, on the other hand, which is getting benefit of the fact that the output nonlinearities are locally invertible due to their polynomial nature. The first approach, commonly called the over parameterisation method, is characterised by modelling the linear dynamic part via an input-output ARMAX mathematical model with unknown time-varying parameters. The formulation of the parametric estimation problem is conducted by a new adaptation based on the prediction error method and using the recursive approximate maximum likelihood estimator RAML. The second one, named the optimisation approach, is crucial to avoid the over-sizing and the parameters redundancy problem of the studied model that was used for describing the linear part of the considered system. In this instance, the representation model is transformed in the GOBF. The recent idea is to provide a guess of the optimal GOBF and polynomials models parameters by the recursive extended least squares algorithm (RELS) based on the prediction error method. Adding to that, the RAML and the RELS estimators are combined with two main tools. The first is the singular values decomposition (SVD) technique, so as to separate the coupled parameters. Whereas the second, named as fuzzy technique, is utilised to both determine the variable forgetting factor and ameliorate the estimation quality.

The remainder of this paper is structured as follows. The second section describes the dynamical nonlinear stochastic system by a discrete-time Hammerstein-Wiener structure. The third section deals with the over parameterisation method. The optimisation approach is presented in the fourth section. In Section 5, we provide an illustrative simulation study of the hydraulic process to prove the validity of the developed theoretical approaches. Conclusions are drawn at the end of the research process.

2 Preliminary study of Hammerstein-Wiener stochastic structure

The modelling of stochastic discrete-time nonlinear system can be conducted using the Hammerstein-Wiener structure, which consists of a dynamic linear block incorporated between two static nonlinear ones. Figure 1 illustrates the structure of this model type, with the combination of the three blocks.

Figure 1 Hammerstein-Wiener structure



Based on Figure 1, the considered structure can be expressed by the following mathematical model.

In this study, a derivation of a polynomial function can describe the first nonlinear static block as:

$$h(k) = f_1(u(k)) = \sum_{v=1}^{p} \chi_v u^v(k) + \delta_1(k) \tag{1}$$

where $u(k)$ represents the system input and $h(k)$ is unmeasurable intermediate variable. Besides, $\delta_1(k)$ which represents an approximation error of the nonlinear function, χ_v, $v = 1, \ldots, p$ is an unknown parameter and p indicates the degree of the nonlinearity.

The dynamic linear block can be defined by the following expression:

$$x(k) = G[h(k)] + v(k) \tag{2}$$

wherein $x(k)$ is unmeasurable intermediate variable, $G[\cdot]$ is a linear function and $v(k)$ is a noise sequence representing the perturbation that can act on the system which can be necessary for an accurate modelling.

The second nonlinear block is expressed by the polynomial nonlinear function:

$$y(k) = f_2(x(k)) = \sum_{z=1}^{r} \beta_z x^z(k) + \delta_2(k) \tag{3}$$

where $y(k)$ depicts the system output, β_z, $z = 1, \ldots, r$ is an unknown parameter and r indicates the nonlinearity degree.

Remark 1: It is worth noting that the approximated errors values $\delta_1(k)$ and $\delta_2(k)$ are related to the selection of the nonlinearity degrees p and r, respectively.

Henceforth, in case the values of the approximated errors $\delta_1(k)$ and $\delta_2(k)$ are decreased, when the values of the nonlinearity degrees p and r are increased (Kamoun, 2013; Elloumi and Kamoun, 2016a). In this context, we presume that the selection of the nonlinearity degree p like r is created in a way that the approximated errors $\delta_1(k)$ and $\delta_2(k)$ can be neglected.

According to the remark given above, we can re-define the representation of the mathematical models structures as follows:

$$y(k) = f_2(x(k)) = \sum_{z=1}^{p} \beta_z x^z(k) \tag{4}$$

and

$$h(k) = f_1(u(k)) = \sum_{v=1}^{p} \chi_v u^v(k) \tag{5}$$

This represents a Hammerstein-Wiener model structure. It is particularly general in that it allows for a correlated noise term $v(k)$ preceding the Wiener nonlinearity.

The objective of this paper is to develop new approaches, over parameterisation and optimisation approaches, for identifying Hammerstein-Wiener model. These approaches will develop in the next sections.

3 Over-parameterisation approach

The central aspect employs two main tools. The first is the description of the linear function $G[\cdot]$, shown in equation (2), by the following mathematical model:

$$x(k) = \frac{B\left(q^{-1}, k\right)}{A\left(q^{-1}, k\right)} h(k) + v(k) \tag{6}$$

withal $A(q^{-1}, k)$ and $B(q^{-1}, k)$ are polynomial expressions, which are defined by:

$$A\left(q^{-1}, k\right) = 1 + a_1(k)q^{-1} + \ldots + a_{n_A}(k)q^{-n_A} \tag{7}$$

$$B\left(q^{-1}, k\right) = b_1(k)q^{-1} + \ldots + b_{n_B}(k)q^{-n_B} \tag{8}$$

The second, attached to the representation of the noise $v(k)$ by a moving average MA mathematical model, makes it possible to rewrite the resulting, equation (6), in the following ARMAX mathematical model with unknown time-varying parameters, such as:

$$x(k) = \frac{B\left(q^{-1}, k\right)}{A\left(q^{-1}, k\right)} h(k) + \frac{C\left(q^{-1}\right)}{A\left(q^{-1}, k\right)} e(k) \tag{9}$$

where $\{e(k)\}$ represents the noise sequence, which is considered independent random variables with zero mean and constant variance σ^2 and $C(q^{-1})$ is polynomial expression, which is explained by:

$$C\left(q^{-1}\right) = 1 + c_1 q^{-1} + \ldots + c_{n_C} q^{-n_C} \tag{10}$$

in which n_A, n_B and n_C express the orders of the polynomials.

Otherwise

$$\begin{aligned} x(k) = &-\sum_{j=1}^{n_A} a_j(k)x(k-j) + \sum_{i=1}^{n_B} b_i(k)h(k-i) \\ &+ \sum_{l=0}^{n_C} c_l(k)e(k-l) \end{aligned} \tag{11}$$

Therefore, inserting equation (5) in (11), the intermediate variable $x(k)$ can be expressed by the following developed form:

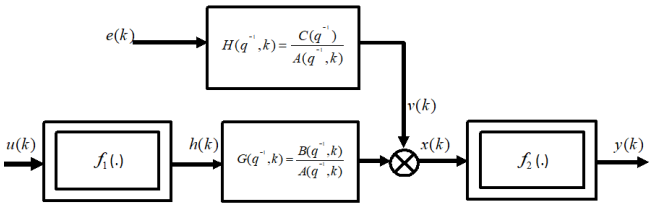
$$\begin{aligned} x(k) = &-\sum_{j=1}^{n_A} a_j(k)x(k-j) + \sum_{i=1}^{n_B} b_i(k) \sum_{v=1}^{p} \chi_v u^v(k-i) \\ &+ \sum_{l=0}^{n_C} c_l(k)e(k-l) \end{aligned} \tag{12}$$

Using equation (12), we can deduce the system output:

$$\begin{aligned} y(k) = \sum_{z=1}^{r} \beta_z x^z(k) = \sum_{z=1}^{r} \beta_z \Bigg(&-\sum_{j=1}^{n_A} a_j(k)x(k-j) \\ &+ \sum_{i=1}^{n_B} \sum_{v=1}^{p} b_i(k)\chi_v u^v(k-i) + \sum_{l=0}^{n_C} c_l(k)e(k-l) \Bigg)^z \end{aligned} \tag{13}$$

The structure of the discrete-time Hammerstein-Wiener mathematical model is presented in Figure 2.

Figure 2 Structure of the discrete-time Hammerstein-Wiener model



After concluding the modelling, we can make the identification of the considered model using a recursive algorithm based on available input and output data $u(k)$, $x(k)$ and $y(k)$. Through examining the previous model (13), we can observe three points causing difficulties in the identification implementation. The first point concerns the nonlinearity of the mathematical model relative to the unknown parameters $\beta_z a_j(k)$, $\beta_z b_i(k)\chi_v$ and $\beta_z c_l$.

The second point, specific to the presence of unavailable internal variable, acts between the linear and the second nonlinear block, which makes it impossible for the parameter estimation to be performed directly on the basis of the input-output data. Adding that, the cascade connection of three blocks automatically reinforces also the coupling problem between the different parameters. Therefore, to neutralise the previous problems, we will suggest, for achieving the unaffectedness of the identification of Hammerstein-Wiener model, two assumptions:

Assumption 1: Concerning the nature of the second nonlinear function f_2 that is bijective. From this assumption, it follows immediately that an inverse function can be implied through the nonlinear function f_2^{-1}, which is expressed on the same domain as the output $y(k)$.

Assumption 2: For reasons of simplicity in the implementation of the parametric estimation, it is suggested that the parameter $\mu_1(k)$, involved in the discrete-time Hammerstein-Wiener model, is well-known at each discrete-time k, such as $\mu_1(k) = 1$, $\forall k$.

According to the first assumption, which is revealed equation (14), the intermediate variable $x(k)$ can be expressed by the equation below:

$$x(k) = f_2^{-1}(y(k)) = \sum_{n=1}^{r} \mu_n y^n(k) \tag{14}$$

Inserting equation (14) in (12), we can rewrite the system output as follows:

$$\begin{aligned} f_2^{-1}(y(k)) = &\sum_{j=1}^{n_A} a_j(k) f_2^{-1}(y(k-j)) \\ &+ \sum_{i=1}^{n_B} b_i(k) \sum_{v=1}^{p} \chi_v u_v(k-v) + \sum_{l=0}^{n_C} c_l e(k-l) \end{aligned} \tag{15}$$

which can be put in the following form:

$$\begin{aligned} \sum_{n=1}^{r} \mu_n y^n(k) = &\sum_{n=1}^{n_A} a_j(k) \sum_{n=1}^{r} \mu_n y^n(k-j) \\ &+ \sum_{i=1}^{n_B} \sum_{v=1}^{p} b_i(k) \chi_v u^v(k-i) + \sum_{l=0}^{n_C} c_l e(k-l) \end{aligned} \tag{16}$$

Taking into consideration the second assumption, the system output $y(k)$ can be recapitulated as:

$$\begin{aligned} y(k) = &-\sum_{n=2}^{r} \mu_n y^n(k) - \sum_{j=1}^{n_A} a_j(k) \sum_{n=1}^{r} \mu_n y^n(k-j) \\ &+ \sum_{i=1}^{n_B} \sum_{v=1}^{p} b_i(k) \chi_v u^v(k-i) + \sum_{l=1}^{n_C} c_l e(k-l) + e(k) \end{aligned} \tag{17}$$

or equivalently, in the following matrix form:

$$y(k) = \theta^T(k)\psi(k) + e(k) \tag{18}$$

Giving that the parameter vector $\theta(k)$ and the observation vector $\psi(k)$ are provided, respectively, by:

$$\begin{aligned} \theta(k) = \big[&\mu_2, \ldots, \mu_r, a_1(k)\mu_2, \ldots, a_1(k)\mu_r, \ldots \\ &a_{n_A}(k), a_{n_A}(k)\mu_2, \ldots, a_{n_A}(k)\mu_r, b_1(k)\chi_1, \ldots, \\ &b_1(k)\chi_p, b_{n_B}(k)\chi_1, \ldots, b_{n_B}(k)\chi_p, c_1, \ldots, c_{n_C}\big] \end{aligned} \tag{19}$$

and

$$\begin{aligned} \psi(k) = \big[&-y^2(k), \ldots, -y^r(k), -y(k-1), \ldots, y^r(k-1), \\ &-y(k-n_A), \ldots, -y^r(k-n_A), u(k-1), \ldots, \\ &u^p(k-1), u(k-n_B), \ldots, u^p(k-n_B), \\ &e(k-1), \ldots, e(k-n_C)\big] \end{aligned} \tag{20}$$

Without significantly losing generality, the two following notations are initially made about the simplicity of implantation:

$$\begin{aligned} &f_{11}(k) \ldots\ldots f_{1p}(k) = b_1(k)\chi_1 \ldots\ldots b_1(k)\chi_p \\ &f_{n_B 1}(k) \ldots\ldots f_{n_B p}(k) = b_{n_B}(k)\chi_1 \ldots\ldots b_{n_B}(k)\chi_p \end{aligned} \tag{21}$$

Hence, the parameter vector $\theta(k)$ can be redefined as below:

$$\begin{aligned} \theta(k) = \big[&\mu_2, \ldots, \mu_r, a_1(k), a_1(k)\mu_2, \ldots, a_1(k)\mu_r, \ldots, \\ &a_{n_A}(k), a_{n_A}(k)\mu_2, \ldots, a_{n_A}(k)\mu_r, \\ &f_{11}(k), \ldots, f_{1p}(k), f_{n_B 1}(k), \ldots, f_{n_B p}(k), \\ &c_1, \ldots, c_{n_C}\big] \end{aligned} \tag{22}$$

Henceforth, we can make the identification of the considered model using a recursive algorithm based on available input and output data $u(k)$ and $y(k)$.

3.1 Parametric estimation of Hammerstein-Wiener mathematical model

In this section, we propose to rewrite the parametric estimation problem of the nonlinear dynamic system operating in the stochastic environment and described by

the class of Hammerstein-Wiener mathematical model, as given by (17). The formulation of the parametric estimation is based upon the calculation of the estimated parameters intervening in $\hat{\theta}(k)$.

However, the appearance of $v(k)$, the stochastic disturbance, before the final nonlinearity f_2 significantly makes the estimation problem harder. Because it is not easy to obtain a perfect estimation for the noisy system as done in deterministic cases due to the necessity to estimate the parameters vector that parameterises a noise structure modelling difficulty of computing the influence.

In the sequel, we aim at designing an identification method that is able to both provide accurate estimates parameters and to ensure on the basis of the following recursive approximated maximum likelihood, using particle filtering, counting on the prediction error method and measuring N point data measurements of input and output to estimate the parameters vector that parameterises a Hammerstein-Wiener structure modelling (Ljung and Söderström, 1983; Kamoun, 2013; Ma and Liu, 2016).

The RAML estimator is given as:

$$\begin{aligned}\hat{\theta}(k) &= \hat{\theta}(k-1) + P(k)\tilde{\psi}(k)\varepsilon(k)\\ P(k) &= \frac{1}{\lambda(k)}\left[P(k-1) - \frac{P(k-1)\tilde{\psi}(k)\tilde{\psi}^T(k)P(k-1)}{\lambda(k)+\tilde{\psi}^T(k)P(k-1)\tilde{\psi}(k)}\right]\\ \varepsilon(k) &= y(k) - \hat{\theta}^T(k-1)\bar{\psi}(k)\end{aligned} \tag{23}$$

where $\varepsilon(k)$ is the prediction error, $\hat{\theta}(k)$ represents the estimated parameters vector, $\tilde{\psi}(k)$ is a vector of the filtered observations and $\bar{\psi}(k)$ is a vector of the approximated observations vector $\psi(k)$, which are defined by:

$$\begin{aligned}\hat{\theta}(k) = \big[&\hat{\mu}_2(k), \ldots, \hat{\mu}_r(k), \hat{a}_1(k), \hat{a}_1(k)\hat{\mu}_2(k), \ldots,\\ &\hat{a}_1(k)\hat{\mu}_r(k), \ldots, \hat{a}_{n_A}(k), \hat{a}_{n_A}(k)\hat{\mu}_2(k), \ldots,\\ &\hat{a}_{n_A}(k)\hat{\mu}_r(k), \hat{f}_{11}(k), \ldots, \hat{f}_{1p}(k),\\ &\hat{f}_{n_B1}(k), \ldots, \hat{f}_{n_Bp}(k), \hat{c}_1(k), \ldots, \hat{c}_{n_C}(k)\big]\end{aligned} \tag{24}$$

$$\begin{aligned}\bar{\psi}(k) = \big[&-y^2(k), \ldots, -y^r(k), -y(k-1), \ldots, y^r(k-1)\\ &-y(k-n_A), \ldots, -y^r(k-n_A),\\ &u(k-1), \ldots, u^p(k-1), \ldots, u(k-n_B), \ldots,\\ &u^P(k-n_B), \varepsilon(k-1), \ldots, \varepsilon(k-n_c)\big]\end{aligned} \tag{25}$$

and

$$\begin{aligned}\tilde{\psi}(k) = \big[&-\tilde{y}^2(k), \ldots, -\tilde{y}^r(k), -\tilde{y}(k-1), \ldots,\\ &-\tilde{y}^r(k-1), -y(k-n_A), \ldots, -\tilde{y}^r(k-n_A),\\ &\tilde{u}(k-1), \ldots, \tilde{u}^p(k-1), \ldots, \tilde{u}(k-n_B), \ldots,\\ &\tilde{u}^P(k-n_B), \tilde{\varepsilon}(k-1), \ldots, \tilde{\varepsilon}(k-n_C)\big]\end{aligned} \tag{26}$$

where $\lambda(k)$ is a forgetting factor and $P(k)$ is a matrix of adaptation gain.

Furthermore, the filtered signals at the discrete-time k, $\tilde{y}(k)$, $\tilde{u}(k)$ and $\tilde{\psi}(k)$, are provided by:

$$\tilde{y}(k) = \frac{y(k)}{\hat{C}(q^{-1}, k-1)} \tag{27}$$

$$\tilde{\psi}(k) = \frac{\bar{\psi}(k)}{\hat{C}(q^{-1}, k-1)} \tag{28}$$

$$\tilde{u}(k) = \frac{u(k)}{\hat{C}(q^{-1}, k-1)} \tag{29}$$

3.2 SVD and fuzzy techniques

The main difficulties of this parametric estimation are to extract directly the estimated values, $\hat{b}_i(k)$ and $\hat{\chi}_v$, and to improve the estimations quality. Consequently, this estimator, RAML, is combined with a specific decomposition technique, the SVD (Zhang et al., 2006), based on the values of the estimated $\tilde{f}_{iv}(k)$ to deduce the separated parameters. Besides, a fuzzy technique is used to determine the factor forgetting variable.

3.2.1 Singular value decomposition

It remains now to separate the parameters of the linear part from those of the nonlinear block. In this respect, to estimate separately the parameters b_i and χ_v, we use the SVD.

The proposed method is based on calculating the estimates $\hat{b}^T = [\hat{b}_1, \ldots, \hat{b}_{n_B}]$ and $\hat{\chi}^T = [\hat{\chi}_1, \ldots, \hat{\chi}_p]$ by the SVD, minimising the following criterion:

$$(\hat{\chi}, \hat{b}) = \arg\min_{\chi, b}\left\{\left\|\hat{\theta}_{\chi b} - \chi b^T\right\|_2^2\right\} \tag{30}$$

Minimising equation (30) is equivalent to resolve:

$$\hat{\theta}_{\chi b} = P_k F_k E_k^T = \sum_{j=1}^{k}\sigma_j p_j e_j^T = [P_1 P_2]\begin{pmatrix} F_1 & 0\\ 0 & F_2\end{pmatrix}\begin{pmatrix} E_1^T\\ E_2^T\end{pmatrix} \tag{31}$$

where F_k is a diagonal matrix including the k non-zero singular values ($\sigma_i = i, \ldots, k$) of $\hat{\theta}_{\chi b}$, the matrixes P_k and E_k contain only the first k columns of the unitary matrices P and E. Therefore, the estimation of the separated parameters are defined by the following equations:

$$\hat{\chi} = P_1 \tag{32}$$

$$\hat{b} = E_1 F_1 \tag{33}$$

3.2.2 Fuzzy technique

The function of the forgetting factor $\lambda(k)$ is to enhance the capacity of the matrix adaptation gain $P(k)$ while supplying a better tracking of the constant and time-varying parameters of the considered system. The selection of the variable forgetting factor has to be made in a suitable way, it can be chosen constant or time-varying depending on the type of the considered application. Therefore, it is obvious that the objective of the choice of this time-varying

forgetting factor $\lambda(k)$ is to better refine the estimation quality that affects the capacity performances of the identification. Let us emphasise the range of variation of forgetting factor under the condition $(0 \prec \lambda(k) \prec 1)$.

So that, in order to improve the estimations level, we propose to compute the variable forgetting factor by using a fuzzy supervisor technique, which checks the prediction error and chooses the suitable forgetting factor at each discrete time k. For convenience, let us recapitulate the forgetting factor relationships obtained by using simulation studies. The establishment of the fuzzy rules are defined as:

- if $|\varepsilon(k)|$ is small, so, $\lambda(k)$ is large
- if $|\varepsilon(k)|$ is medium, so, $\lambda(k)$ is medium
- if $|\varepsilon(k)|$ is large, so, $\lambda(k)$ is small.

Remark 2: It is worth noting that the shortcomings of this approach are summarised in the redundancy problem between the different parameters of the Hammerstein-Wiener model resulted from the representation of the linear dynamic element and the correlated noise, which automatically reinforces the over-sizing of the model. Hence, in the next section, a simple but effective solution to this problem is introduced for getting the benefit from the Hammerstein-Wiener model.

4 Optimisation approach

The key of this approach is to represent the dynamic linear part and the noise of the Hammerstein-Wiener model by the best linear approximation namely the orthonormal basis filter (OBF). Now, it is obvious that OBF-based model involves qualities that are highly attractive to capture linear dynamics. One of its crucial reasons is the fact that the OBF model permits one or more poles of the system to be introduced in the model which renders the OBF model in grey nature. But in this work, the main idea for utilising OBF is to decrease the dimension of the unknown parameters matrix.

One of the major steps in OBF model development is the selection of an appropriate type of orthonormal basis filter. There are various types of OBF which are discussed below.

The Laguerre filters are first-order filters that contain one real pole (Li and Shi, 2011). They can be expressed by:

$$f_i(q) = \sqrt{(1-p)^2}\,\frac{(1-pq)^{i-1}}{(q-p)^i} \quad |p| \prec 1 \tag{34}$$

Kautz filters allow the incorporation of a pair of conjugate complex poles, which can be described as:

$$\begin{aligned} f_{2i-1}(q) &= \frac{\sqrt{(1-a)^2(1-b)^2}}{q^2+a(b-1)(q-b)}\,g(a,b,q,i) \\ f_{2i}(q) &= \frac{\sqrt{(1-b)^2(q-a)}}{q^2+a(b-1)(q-b)}\,g(a,b,q,i) \end{aligned} \tag{35}$$

$$g(q,b,i) = \left(\frac{bq^2+a(b-1)q+1}{q^2+a(b-1)-b}\right)^{i-1} \tag{36}$$

$-1 \prec a \prec 1,\ -1 \prec b \prec 1$ and $i = 1, 2, \ldots$

The main disadvantage of these two preceding OBF types is that one or two poles can only be introduced into each Laguerre and Kautz function, respectively, which will not be able to describe the system dynamics. As a solution, Akçay and Ninness (1999) developed the generalised orthonormal basis functions (GOBF).

Recall that, the GOBFs are defined as:

$$L_j(q) = \frac{\sqrt{\left(1-|p_j|^2\right)}}{(q-p_j)}\prod_{i=0}^{j-1}\frac{\left(1-\overline{p_i}q\right)}{(q-p_i)} \tag{37}$$

where $p = \{p_i, p_j\}$ is a sequence of poles inside the unit circle appearing in complex conjugate pairs.

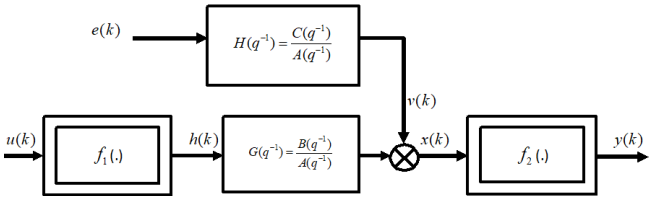
According to the schematic structure of the Hammerstein-Wiener model given by Figure 2, in this section, the linear dynamic part and the modelled noise of the considered model are represented by both linear time-invariant systems $G(q^{-1})$ and $H(q^{-1})$, respectively, which are parameterised as rational forms in q^{-1}:

$$H\left(q^{-1}\right) = \frac{C\left(q^{-1}\right)}{A\left(q^{-1}\right)} \tag{38}$$

$$G\left(q^{-1}\right) = \frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)} \tag{39}$$

Therefore, Figure 3 introduced the newer representation of the Hammerstein-Wiener model.

Figure 3 A discrete-time Hammerstein-Wiener model



Assumption 3: the LTI systems, $G(q^{-1})$ and $H(q^{-1})$, are restricted to be a stable rational function. Yet means that their poles, are denoted by p_i and p_j, respectively, inside the unit circle.

During the transformation on the GOBF, the equations can be rewritten in the form below:

$$G\left(q^{-1}\right) = \sum_{j=0}^{n-1} d_j L_j\left(q^{-1}\right) \tag{40}$$

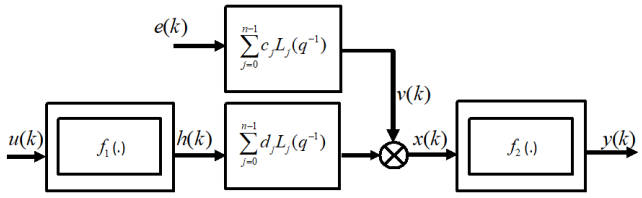
$$H\left(q^{-1}\right) = \sum_{j=0}^{n-1} c_j L_j\left(q^{-1}\right) \tag{41}$$

where n represents the number of the poles and $L_j(q^{-1})$ illustrates the GOBFs. In which p_i and $\overline{p_i}$ represent,

respectively, the pole and its conjugate, and q^{-1} back shift operator.

We can illustrate the block diagram of Hammerstein-Wiener model by Figure 4.

Figure 4 Structure of Hammerstein-Wiener mathematical model based on GOBF



Based on Figure 4 and the equations (5) and (9), the intermediate unmeasurable variable can be expressed as follows:

$$x(k)=\sum_{j=0}^{n-1}\sum_{v=1}^{p} d_j \chi_v L_j\left(q^{-1}\right)u^v(k)+\sum_{j=0}^{n-1} c_j L_j\left(q^{-1}\right)e(k) \quad (42)$$

Taking into consideration the equation (42) and the assumptions cited in the second section, the system output $y(k)$ can be provided as:

$$\begin{aligned} y(k)=&-\sum_{n=2}^{r}\mu_n y^n(k)+\sum_{j=0}^{n-1}\sum_{v=1}^{p} d_j \chi_v L_j\left(q^{-1}\right)u^v(k)\\ &+\sum_{j=0}^{n-1} c_j L_j\left(q^{-1}\right)e(k)\end{aligned} \quad (43)$$

We can express the system output, equation (43), as follows:

$$y(k)=\theta^T\psi(k) \quad (44)$$

where θ is the parameter vector and $\psi(k)$ represents the observation vector, which are respectively defined by:

$$\begin{aligned}\theta=&\left[\mu_2,\ldots,\mu_r,d_0\chi_1,\ldots,d_0\chi_p,\ldots,\right.\\ &\left.d_{n-1}\chi_1,\ldots,d_{n-1}\chi_p,c_0,\ldots,c_{n-1}\right]\end{aligned} \quad (45)$$

and

$$\begin{aligned}\psi(k)=&\left[-y^2(k),\ldots,-y^r(k),L_0\left(q^{-1}\right)u(k),\ldots,\right.\\ &L_0\left(q^{-1}\right)u^P(k),\ldots,L_{n-1}\left(q^{-1}\right)u(k),\ldots,\\ &L_{n-1}\left(q^{-1}\right)u^P(k),L_0\left(q^{-1}\right)e(k),\ldots,\\ &\left.L_{n-1}\left(q^{-1}\right)e(k)\right]\end{aligned} \quad (46)$$

As the second section is for parsimony of approximation, we propose to use the two following notations:

$$\begin{aligned} g_{01}\ldots\ldots g_{0p}&=d_0\chi_1\ldots\ldots d_0\chi_p\\ g_{n-11}\ldots\ldots g_{n-1p}&=d_{n-1}\chi_1\ldots\ldots d_{n-1}\chi_p\end{aligned} \quad (47)$$

Under this notation, the combined linear, nonlinear, and noise descriptions are therefore parameterised by the parameter vector θ, which can be specified as follows:

$$\begin{aligned}\theta=&\left[\mu_2,\ldots,\mu_r,g_{01},\ldots,g_{0p},\ldots,\right.\\ &\left.g_{n-11},\ldots,g_{n-1p},c_0,\ldots,g_{n-1}\right]\end{aligned} \quad (48)$$

In order to obtain estimated parameters with a minimum variance, we can affirm that the formulation of a parametric estimation scheme presents serious difficulties. Firstly, we estimate the parameters vector θ starting from the knowledge of the quantities intervening in the observation vector $\psi(k)$.

Nonetheless, the observation vector is not totally observable since it contains the noise sequence $\{e(k-t), t=1,\ldots,n\}$ which is not measurable.

For the purpose of overcoming this parametric estimation problem, we introduce the RELS, recalling on the prediction error method under the approximation $\bar{\psi}(k)$ of this vector (Billings and Voon, 1984; Elloumi and Kamoun, 2016a), such as:

$$\begin{aligned}\bar{\psi}(k)=&\left[-y^2(k),\ldots,-y^r(k),\right.\\ &L_0\left(q^{-1}\right)u(k),\ldots,L_0\left(q^{-1}\right)u^P(k),\\ &L_{n-1}\left(q^{-1}\right)u(k),\ldots,L_{n-1}\left(q^{-1}\right)u^P(k),\\ &\left.L_0\left(q^{-1}\right)\varepsilon(k),\ldots,L_{n-1}\left(q^{-1}\right)\varepsilon(k)\right]\end{aligned} \quad (49)$$

The required estimator can be done as follows:

$$\begin{aligned}\hat{\theta}(k)&=\hat{\theta}(k-1)+P(k)\bar{\psi}(k)\varepsilon(k)\\ P(k)&=\frac{1}{\lambda(k)}\left[P(k-1)-\frac{P(k-1)\bar{\psi}(k)\bar{\psi}^T(k)P(k-1)}{\lambda(k)+\bar{\psi}(k)P(k-1)\bar{\psi}(k)}\right]\\ \varepsilon(k)&=y(k)-\hat{\theta}^T(k-1)\bar{\psi}(k)\end{aligned} \quad (50)$$

where $\hat{\theta}(k)$ represents the estimated parameters vector which is shown by:

$$\begin{aligned}\hat{\theta}(k)=&\left[\hat{\mu}_2(k),\ldots,\hat{\mu}_r(k),\hat{g}_{01}(k),\ldots,\hat{g}_{0p}(k),\right.\\ &\left.\hat{g}_{n-11}(k),\ldots,\hat{g}_{n-1p}(k),\hat{c}_0(k),\ldots,\hat{c}_{n-1}(k)\right]\end{aligned} \quad (51)$$

Remark 3: In the same way as RAML estimator, the RELS estimator is incorporated with the SVD technique, to estimate the separated parameters, $\hat{d}^T=[\hat{d}_0,\ldots,\hat{d}_{n-1}]$ and $\hat{\chi}^T=[\hat{\chi}_1,\ldots,\hat{\chi}_p]$, based on the entire coupled estimated parameters data $\hat{g}_{n-1p}(k)$.

Thus, the fuzzy supervisor is integrated into the recursive algorithm for parametric estimation RELS (50) to be exploited in an optimal way and among several values in every moment, the appropriate forgetting factor according to the criterion based on the prediction error.

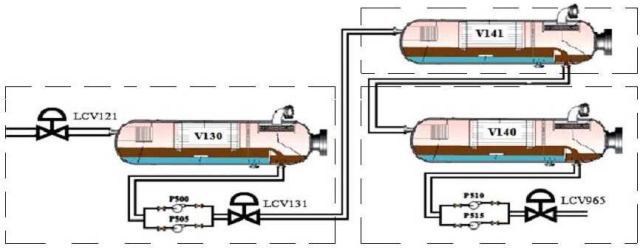
For a preview of these notes, in the next section, a nonlinear hydraulic system study is presented to illustrate the efficiency of the proposed approaches in the identification analysis.

5 Simulation study

In this section, we achieve an illustration simulation in order to test the performance and the efficiency of the developed approaches. In this case, we consider a nonlinear hydraulic process which is composed of three separators (V130, V140 and V141), four motor-pumps shipments (P500, P505 and P510, P515) and three valves control (LCV121, LCV131 and LCV965), as depicted in Figure 5. The role of these separators is to treat the fluid as it comes from other separation stage, to separate the three phases: water, oil and gas, with stabilisation of the oil level in each one. The mathematical model of the considered process is obtained on the basis of theoretical analysis, given by Elloumi and Kamoun (2016a) as follows:

$$\begin{aligned}\frac{dH_1(t)}{dt} &= 0.147Q_e(t) \\ &\quad - C_v(t)\sqrt{126.3851H_1(t)+9{,}180.5} \\ \frac{dH_2(t)}{dt} &= C_v(t)\sqrt{105.041H_1(t)+7{,}630.14} \\ &\quad - 0.1028\sqrt{H_2(t)} \\ \frac{dH_3(t)}{dt} &= 0.005\sqrt{H_3(t)} \\ &\quad - C_{v3}(t)\sqrt{32.656H_3(t)+1{,}471.82}\end{aligned} \tag{52}$$

Figure 5 Oil circuit diagram of the hydraulic process (see online version for colours)



To estimate the parameters of the considered hydraulic system, we must proceed with the following steps:

Step 1 Describe the dynamic of our process by a discrete time Hammerstein-Wiener mathematical model with unknown time-varying parameters, as given by equation (17), in such a way that:

We assume that $n_A = n_B = 2$, $n_C = 1$, $r = 3$ and $p = 2$.

$$\begin{aligned}y(k) &= -\mu_2 y^2(k) - \mu_3 y^3(k) - a_1(k)y(k-1) \\ &\quad - a_1(k)\mu_2 y^2(k-1) - a_1(k)\mu_3 y^3(k-1) \\ &\quad - a_2(k)y(k-2) - a_2(k)\mu_2 y^2(k-2) \\ &\quad - a_2(k)\mu_3 y^3(k-2) + f_{11}(k)u(k-1) \\ &\quad + f_{12}(k)u^2(k-1) + f_{21}(k)u(k-2) \\ &\quad + f_{22}(k)u^2(k-2) + c_1(k)e(k-1) + e(k)\end{aligned} \tag{53}$$

This nonlinear output can be rewritten in the matrix form:

$$y(k) = \theta^T(k)\psi(k) + e(k) \tag{54}$$

Giving that the parameter vector $\theta(k)$ and the observation vector $\psi(k)$ are defined as follows:

$$\begin{aligned}\theta(k) = \big[&\mu_2, \mu_3, a_1(k), a_1(k)\mu_2, a_1(k)\mu_3, \\ &a_2(k), a_2(k)\mu_2, a_2(k)\mu_3, f_{11}(k) \\ &f_{12}(k), f_{21}(k), f_{22}(k), c_1\big]\end{aligned} \tag{55}$$

$$\begin{aligned}\psi(k) = \big[&-y^2(k), -y^3(k), -y(k-1), -y^2(k-1), \\ &-y^3(k-1), -y(k-2), -y^2(k-2), \\ &-y^3(k-2), u(k-1), u^2(k-1), u(k-2), \\ &u^2(k-2), e(k-1)\big]\end{aligned} \tag{56}$$

Step 2 Estimate the parameters of the nonlinear hydraulic system using the RAML algorithm (23) with supervisor forgetting factor.

The relative data to this simulation process are selected as:

- $P(0) = 1{,}000I$, with I is an identity matrix
- the number of measurements M being identified as: $M = 1, ..., 1{,}000$.

By integrating the fuzzy supervisor in the recursive algorithm for parametric estimation RAML (23), we can exploit in an optimal way, at every moment, and among several values, the appropriate forgetting factor according to the criterion based on the prediction error. Figures 6 and 7 show the evolution curves of the membership functions.

Figure 6 Evolution curves of the membership function of $|\varepsilon(k)|$

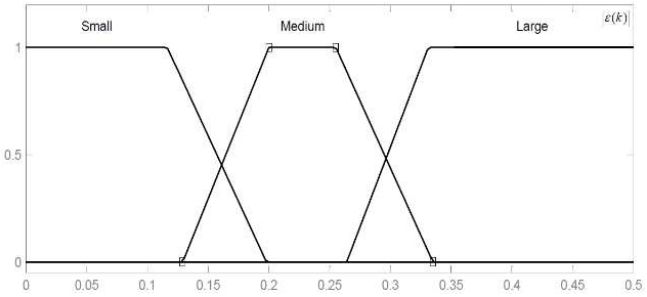
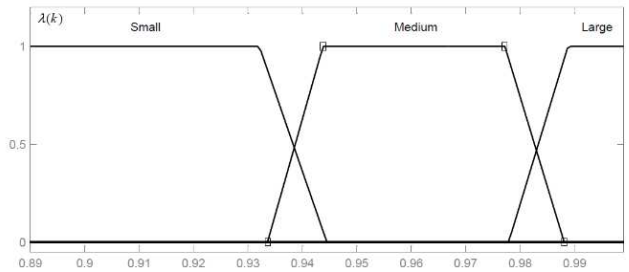


Figure 7 Evolution curves of the membership function of $\lambda(k)$



We illustrate in Figure 8 some simulation results by giving the evolution curves of the simulated and the estimated output, the prediction error $\varepsilon(k)$, its

overall variance $\sigma^2_{\varepsilon(k)}$ and the estimated parameter $\hat{c}_1(k)$.

where

$$\overline{m}_{x(k)} = \frac{\sum_{k=401}^{1,000} x(k)}{600} \tag{57}$$

and

$$\sigma_x^2 = \frac{\sum_{k=401}^{1,000} \left[x(k) - \overline{m}_{x(k)}\right]^2}{600} \tag{58}$$

In addition, the evolution curves of the estimated parameters of the Hammerstein-Wiener model are illustrated in Figures 9, 10 and 11.

Remark 4: The purpose of this numerical simulation illustration is to check the efficiency and performance of the RAML algorithm. Figure 12 represents the evolution curves of the prediction error $\varepsilon(k)$ and its overall variance $\sigma^2_{\varepsilon(k)}$ by using the RAML with constant forgetting factor $\lambda(k) = 0.984$ in order to compare it with the simulation results, the prediction error $\varepsilon(k)$ and its overall variance $\sigma^2_{\varepsilon(k)}$ as presented on Figure 8, where we utilised the RAML with supervisor forgetting factor.

We can affirm that the obtained results, during this example of the numerical simulation, are demonstrated that the recursive parametric estimation algorithm RAML (23), based on the supervisor forgetting factor is satisfactory and gives better performances than the RAML with constant forgotten factor.

Step 3 Separate the coupled parameters, which are estimated in the second step using the SVD technique. After that, we determine the poles of the considered process. The statistical average values of the various estimated parameters are pointed out in Tables 1 and 2.

Table 1 Statistical averages values of the parameters

Parameters	$\hat{m}$	*Parameters*	$\hat{m}$
$\hat{\mu}_2(k)$	–0.5216	$\hat{a}_1(k)$	–1.1083
$\hat{\mu}_3(k)$	0.0647	$\hat{a}_2(k)$	0.1105
$\hat{c}_1(k)$	0.0221	$\sigma^2_{\hat{\varepsilon}}$	0.00001

Table 2 Statistical averages values of the estimated coupled and separated parameters

Parameters	$\hat{m}$	*Parameters*	$\hat{m}$
$\hat{f}_{11}(k)$	0.2102	$\hat{b}_1(k)$	–0.3556
$\hat{f}_{21}(k)$	–0.0647	$\hat{b}_2(k)$	0.1247
$\hat{f}_{12}(k)$	–0.2869	$\hat{\chi}_1(k)$	–0.5832
$\hat{f}_{22}(k)$	0.1070	$\hat{\chi}_2(k)$	0.8123

Step 4 Construct the generalised orthonormal basis, where $n = 2$, $p_0 = 0.9976$ and $p_1 = 0.1107$.

$$\begin{aligned} L_0(q) &= \frac{\sqrt{\left(1-|p_0|^2\right)}}{(q-p_0)} \\ L_1(q) &= \frac{\sqrt{\left(1-|p_1|^2\right)}}{(q-p_1)} \frac{\left(1-\overline{p_0}q\right)}{(q-p_0)} \end{aligned} \tag{59}$$

Recall that, the process modelling can be represented by the Hammerstein-Wiener mathematical model as provided by equation (43).

Therefore, the system output can be developed in the following form:

$$\begin{aligned} y(k) = &-\mu_2 y^2(k) - \mu_3 y^3(k) + g_{01} L_0\left(q^{-1}\right) u(k) \\ &+ g_{02} L_0\left(q^{-1}\right) u^2(k) + g_{11} L_1\left(q^{-1}\right) u(k) \\ &+ g_{12} L_1\left(q^{-1}\right) u^2(k) + c_0 L_0\left(q^{-1}\right) e(k) \\ &+ c_1 L_1\left(q^{-1}\right) e(k) \end{aligned} \tag{60}$$

or equivalently in matrix form:

$$y(k) = \theta^T \psi(k) \tag{61}$$

where θ and $\psi(k)$ are, respectively, the parameter vector and the observation vector, which are defined by:

$$\theta = \left[\mu_2, \mu_3, g_{01}, g_{02}, g_{11}, g_{12}, c_1, c_2\right] \tag{62}$$

and

$$\begin{aligned} \psi(k) = \big[&-y^2(k), -y^3(k), \\ &L_0\left(q^{-1}\right) u(k),\ L_0\left(q^{-1}\right) u^2(k), \\ &L_1\left(q^{-1}\right) u(k), L_1\left(q^{-1}\right) u^2(k), \\ &L_2\left(q^{-1}\right) e(k), L_1\left(q^{-1}\right) e(k)\big] \end{aligned} \tag{63}$$

Figure 8 Evolution curves of $y(k)$, $\hat{y}(k)$, $\varepsilon(k)$, $\sigma^2_{\varepsilon(k)}$ and $\hat{c}_1(k)$ (see online version for colours)

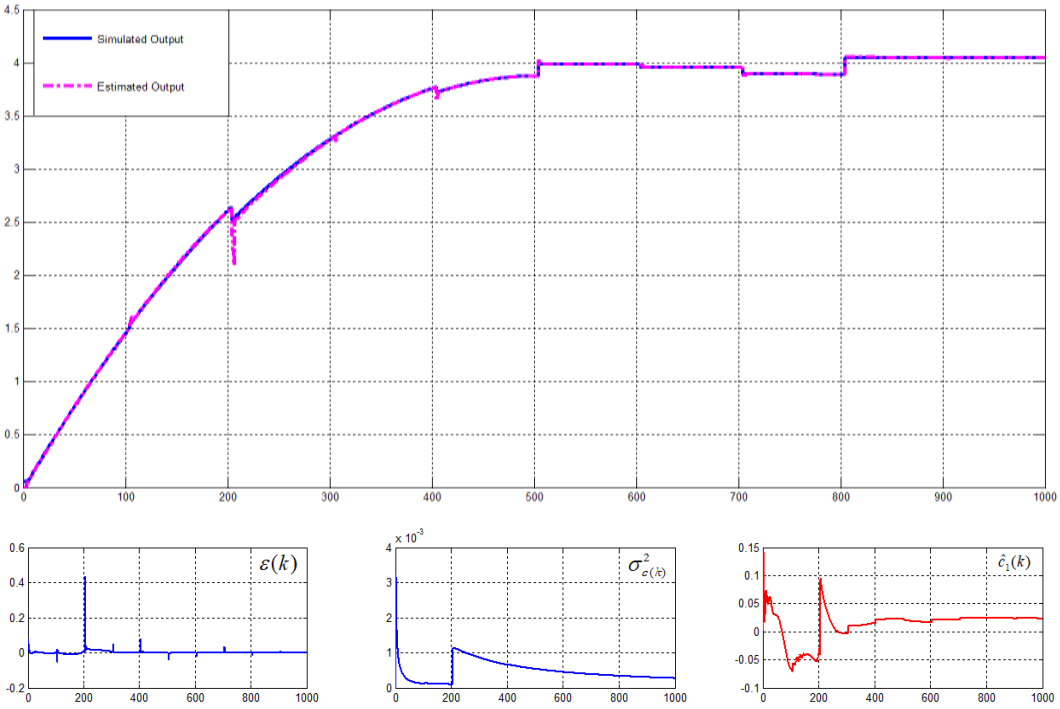


Figure 9 Evolution curves of the estimated parameters $\hat{\mu}_2(k)$, $\hat{\mu}_3(k)$, $\hat{a}_1(k)$ and $\hat{a}_1(k)\hat{\mu}_2(k)$ (see online version for colours)

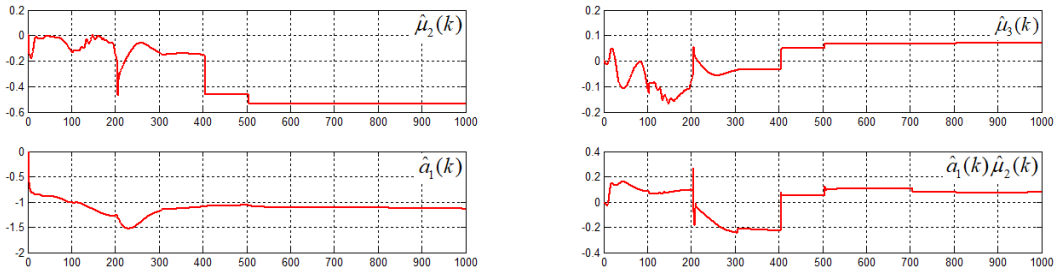


Figure 10 Evolution curves of $\hat{a}_1(k)\hat{\mu}_3(k)$, $\hat{a}_2(k)$, $\hat{a}_2(k)\hat{\mu}_2(k)$ and $\hat{a}_2(k)\hat{\mu}_3(k)$ (see online version for colours)

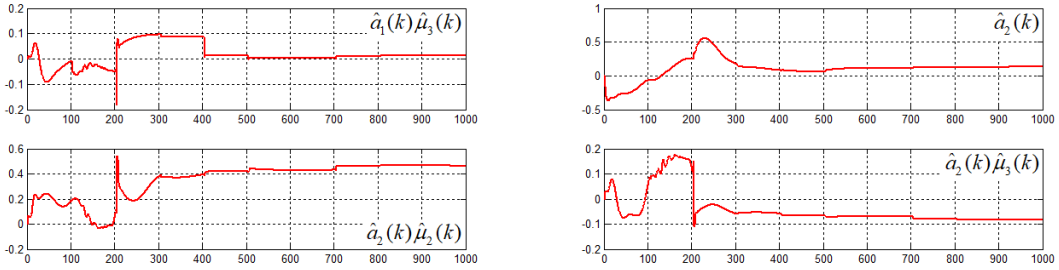


Figure 11 Evolution curves of $\hat{f}_{11}(k)$, $\hat{f}_{12}(k)$, $\hat{f}_{21}(k)$ and $\hat{f}_{22}(k)$ (see online version for colours)

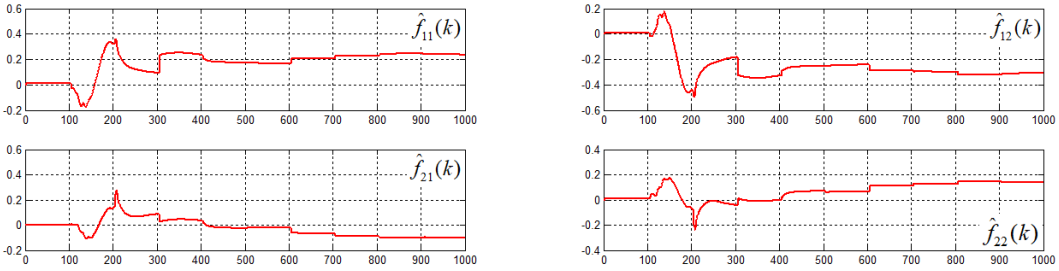


Figure 12 Evolution curves of $\varepsilon(k)$ and $\sigma^2_{\varepsilon(k)}$ (see online version for colours)

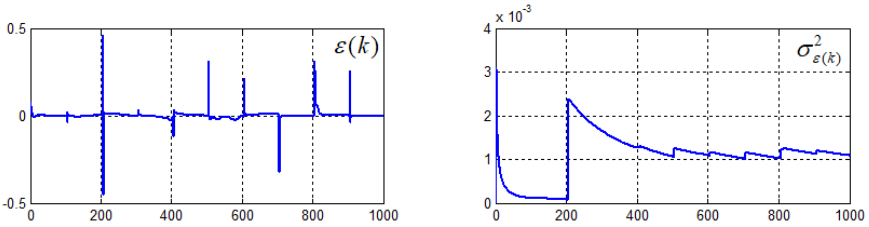


Figure 13 Evolution curves of $y(k)$, $\hat{y}(k)$, $\varepsilon(k)$ and $\sigma^2_{\varepsilon(k)}$ (see online version for colours)

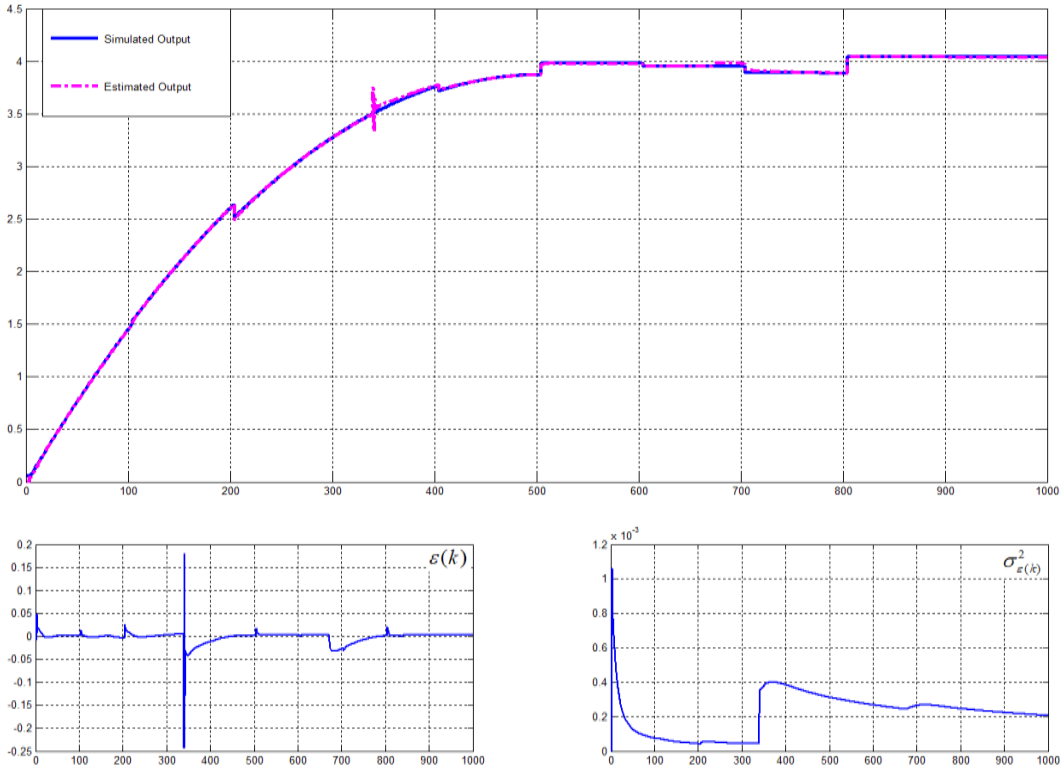


Figure 14 Evolution curves of the estimated parameters $\hat{\mu}_1(k)$ and $\hat{\mu}_2(k)$ (see online version for colours)

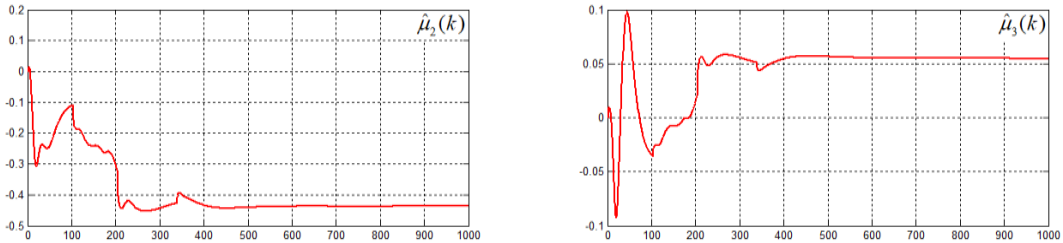


Figure 15 Evolution curves of the estimated parameters $\hat{g}_{01}(k)$ and $\hat{g}_{02}(k)$ (see online version for colours)

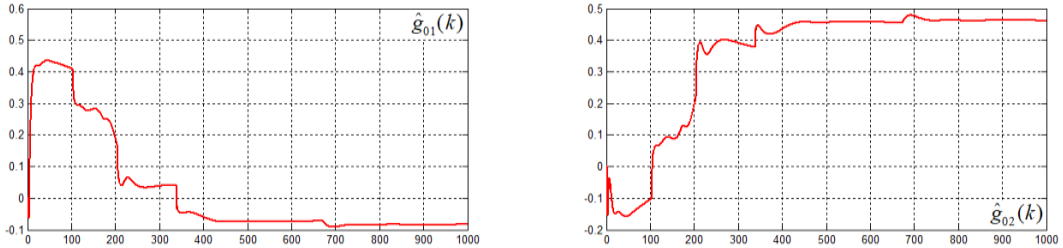


Figure 16 Evolution curves of the estimated parameters $\hat{g}_{11}(k)$ and $\hat{g}_{12}(k)$ (see online version for colours)

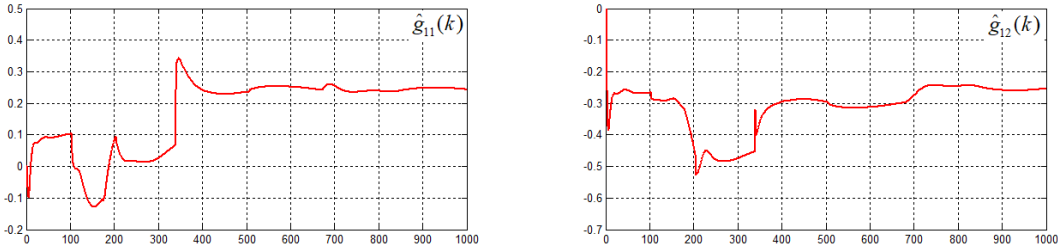
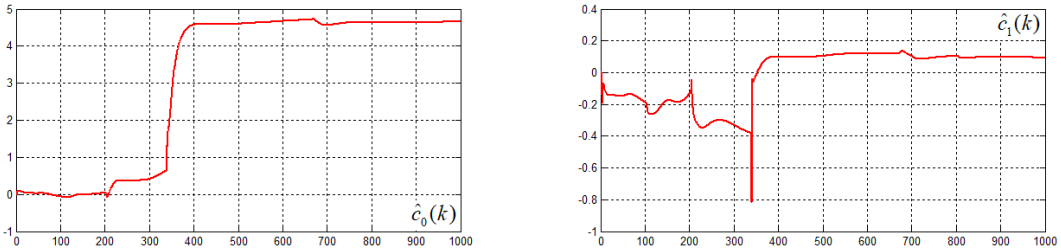


Figure 17 Evolution curves of the estimated parameters $\hat{c}_0(k)$ and $\hat{c}_1(k)$ (see online version for colours)



Step 5 Estimate the parameters intervening in the equation (60) utilising the recursive extended least squares (50) based on the prediction error method.

For this purpose, the prediction error and the variable forgetting factor are chosen, such as: $|\varepsilon(k)| \in [0, 0.3]$ and $\lambda(k) \in [0.9, 1]$.

The evolution curves of the simulated and the estimated output, the prediction error $\varepsilon(k)$ and its overall variance $\sigma^2_{\varepsilon(k)}$ are stated in Figure 13.

Adding that the evolution curves of the estimated parameters are given in Figures 14, 15, 16 and 17.

Utilising the (SVD), the statistical average values of the estimated parameters are presented in Tables 3 and 4. The computation of the overall variance $\sigma^2_{\varepsilon(k)}$ value is performed for $k = 701, \ldots, 1{,}000$.

The obtained results in this illustration simulation show well the performances that are provided by the use of two approaches which can be remarked in the low values of the shapes of prediction errors. Thus, we can affirm that the recursive estimators, RAML (23) and RELS (50), are satisfactory and clearly prove the good performance during the estimate of the nonlinear hydraulic system, which is described by Hammerstein-Wiener mathematical models (53) and (60), respectively. But also different from over parameterisation approach, it is clear that the optimisation approach has another feature that is assimilated to simpler structure and less sizing dimension of the unknown parameters matrix which can simplify the formulation and the implementation of the control strategies by Hammerstein-Wiener model. In the end, the simulation results show that the proposed approaches are numerically robust and effective in the identification of Hammerstein-Wiener model that can be illustrated to other blocks oriented models.

Table 3 Statistical averages values of the estimated parameters

Parameters	$\hat{m}$	*Parameters*	$\hat{m}$
$\hat{\mu}_2(k)$	–0.4371	$\hat{c}_0(k)$	4.6369
$\hat{\mu}_3(k)$	0.0550	$\hat{c}_1(k)$	0.0953
σ^2_ε	0.00028		

Table 4 Statistical averages values of the estimated coupled and separate parameters

Parameters	$\hat{m}$	*Parameters*	$\hat{m}$
$\hat{g}_{01}(k)$	–0.0846	$\hat{d}_0(k)$	0.4614
$\hat{g}_{11}(k)$	0.2427	$\hat{d}_1(k)$	–0.3241
$\hat{g}_{02}(k)$	0.4629	$\hat{\chi}_1(k)$	–0.3702
$\hat{g}_{12}(k)$	–0.2522	$\hat{\chi}_2(k)$	0.9290

6 Conclusions

In this paper, two iterative parametric estimation procedures are developed for a class of nonlinear stochastic system, where its dynamics are described by discrete Hammerstein mathematical models with known structure variables and unknown time-varying parameters. The static nonlinear block is approximated by polynomial functions. In other words, the first approach corresponds to the over

parameterisation method where the dynamic linear part is described by a single-input single-output ARMAX mathematical model with unknown time-varying parameters. As for the second one, it is related to the optimisation approach where the considered process is modelled by the GOBF representation. In these developed approaches, the formulation of the parametric estimation problem is performed based on the prediction error method by using the RAML and the RELS algorithms. It should also be mentioned that these recursive estimators are combined with the SVD, to separate the coupled parameters, and the fuzzy technique to obtain the best estimation quality with the smallest estimation error. Our developed approaches make it possible to estimate the parameters involved in the Hammerstein-Wiener model despite the presence of the redundant parameters, the noise acting on the system output and the great complexity of the model. Nonlinear hydraulic process simulation results are presented to prove the effectiveness and the validity of the developed theoretical procedures.

In future works, we will plan to develop extended versions of different algorithms which permit to estimate the structure variables (order and delay) of the nonlinear systems described by Hammerstein-Wiener structures. It would be also interesting to formulate the control problem by developing several controllers based on different control methods.

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