a change in the heat transfer coefficient). Since the method assumes that the parameters do not vary with operating condition, then the method will only work when the operating condition space is explored over a relatively short time interval. Again, few chemical processes will be operated to produce a product over the wide range used in the example CSTR problem.

Ordinarily, if a chemical process is used to produce a number of products, each at a different operating point, then the operating conditions would be changed through a series of closed-loop setpoint changes. The proposed strategy, however, is to find steady-state operating points based on changing the input (jacket flowrate in the example). Because the process parameters differ from the nominal set of parameters, the actual steady-states may not correspond to useful operating points (where the product produced would presumably have some value). Process plant operators would certainly prefer to move between desired operating points using closed-loop control, particularly when potential long time-scale open-loop dynamics are considered.

The selection of six parameters for estimation is somewhat arbitrary. Indeed, I would argue that parameters 5 and 6, the reactor and jacket feed temperatures, are actually easily measured, and that one of the parameters assumed to be constant, x_{1f} (feed concentration), is more likely to be a disturbance variable. In any event, the approach by Sandlink et al. [6], based on relative gain analysis, may provide a better technique for choosing parameters for estimation. Another minor concern is that there can be a limit to the use of the Euler discretization of the nonlinear model. For some systems it may be better to use a larger sample time and an exact analytical discretization of the linearized model.

3. Summarizing Comments

Particularly as an academic, I hate to appear to be a curmudgeon finding faults with the novel contribution by Collins et al. for estimating a large number of parameters in a model. To confirm the practicality of the proposed approach, I would like to see an example system with a reasonable number of parameters that are constant, but unknown, that can be estimated by changing operating conditions over a set of operating conditions that would be used in practice. Perhaps there are better mechanical or electrical system examples to illustrate the strengths of the proposed stacked model approach.

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Discussion on: "A Stacked Model Structure for Off-line Parameter Variation Estimation in Multi-equilibria Nonlinear Systems"

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Ref. [1] presents a methodology for estimating constant parameters of a nonlinear system, with a specific focus on systems that operate around a finite set of operating points. A continuous stirred tank reactor (CSTR) with nonlinear dynamics is used to demonstrate the approach. The method uses a stacked

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Kalman Filter (KF) or stacked Extended Kalman Filter (EKF) along with data about multiple equilibriumpoints to determine a set of constant parameters. The authors show how a KF does not work well for data about single equilibrium point data because of observability problems; similar issues occur with the EKF. The stacked filter is then used to select the appropriate equilibrium points and to estimate the constant parameters using multiple datasets.

This discussion note focuses on three important points related to parameter estimation in general, and the approaches in Ref. [1] specifically: (a) the KF for parameter estimation, (b) the EKF for parameter estimation, and (c) the Kalman smoother.

1. The Kalman Filter for Parameter Estimation

The use of a KF is only appropriate for systems where the parameters (or states) enter he system linearly. Even for most linear systems, attempting to estimate internal model arameters is a nonlinear estimation problem. Consider the simplest system given by

$$\dot{x} = ax + w \tag{1}$$

If a is constant and known, the KF can be used to develop an optimal estimate of the state $\hat{x}$. But, if a is constant, unknown and must be estimated, the system then becomes

$$\begin{bmatrix} \dot{x} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} ax \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w \tag{2}$$

Using a KF for estimating both x and a will use the wrong linearized system matrix and result in the development of biased estimates.

2. The Extended Kalman Filter for Parameter Estimation

Given a parameter estimation problem with inherently nonlinear dynamics, as is the case in Ref. [1], the appropriate approach is to use a nonlinear parameter estimation scheme such as EKF [2], sigma point filter [3], or similar methods. The use of an EKF for parameter estimation has a rich history [4, 2], with critical applications such as gyro bias estimation [5] and aerodynamic parameter estimation [6]. Extensions have been made by many researchers.

Even the EKF, however, when used to estimate parameters, is known to suffer from bias errors if the model is incorrect. One common example is if the noise matrices are modeled incorrectly [4, 7]. As such, any implementation of the EKF for parameter estimation should evaluate the consistency of the estimator, as well as the confidence bounds of the estimates. Ref. [2] describes several statistical tests to evaluate the accuracy of the EKF, such as a Chi-square test statistic and an evaluation of innovation whiteness.

The final confidence bounds for an EKF parameter estimation technique are a function of a number of factors, including system observability, the number and size of the noises, and the amount of data used. For the parameter estimation problem given in Ref. [1], an EKF run over data about one equilibrium point is fundamentally different in all of these factors when compared with an EKF run over data about four equilibrium points. The four equilibrium point case nicely attempts to estimate one set of parameters using four sets of data in parallel. The four equilibrium point case increases the observability, as Ref. [1] discusses, and includes four times the amount of data; both of these factors decrease the size of the confidence bounds. But there are also four times the amount of process noise.

Fig. 1 presents the time histories for all parameters for an EKF run over data around one equilibriumpoint for the example presented in Ref. [1]. Notice that, even with data about one equilibrium point, all estimates do converge to near the final truth values, and are within the 95% confidence bounds. The system is observable, even about one equilibrium point, which enables the parameter estimates to converge. Compared to the KF, the observability of the EKF for one equilibrium point is improved because the system matrices change at each time step.

Fig. 2 presents the time histories for all parameters for an EKF run over data from four equilibrium points. In this case, the number of data points used in each dataset is one fourth of the case in Fig. 1 in order to make the comparison more appropriate. Comparing the two cases, the trends are similar. The estimates all converge close to the true value, and within the confidence bounds. The estimates from the four equilibrium point case converge more quickly, but this is both a factor of the increased observability and the use of four datasets; the increased convergence rate is less when considering the same amount of data.

3. The Kalman Smoother for Parameter Estimation

For off-line usage, which Ref. [1] is, a Kalman Smoother parameter estimator will yield even better results. The Kalman Smoother runs back and forth over the data until all important information has been

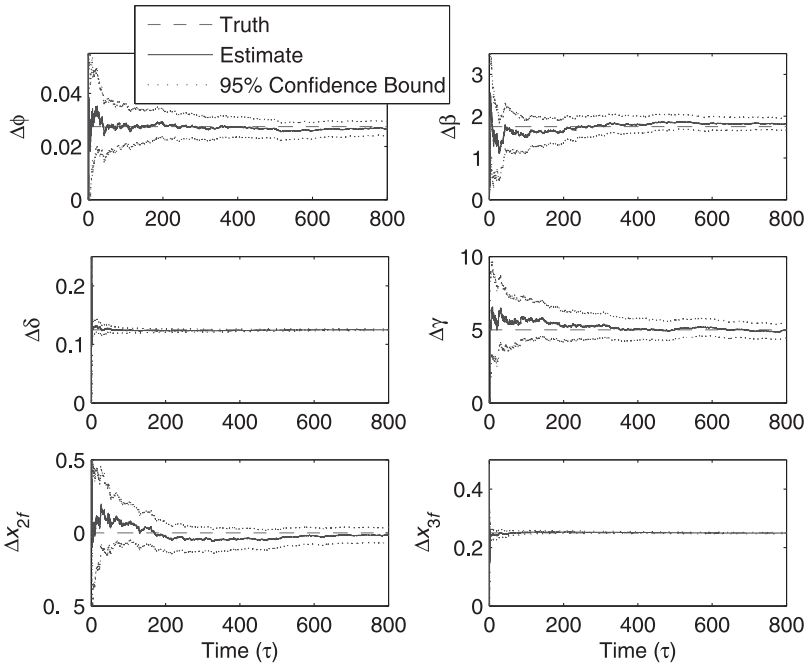


Fig. 1. Parameter estimation history using an EKF and data about one equilibrium point.

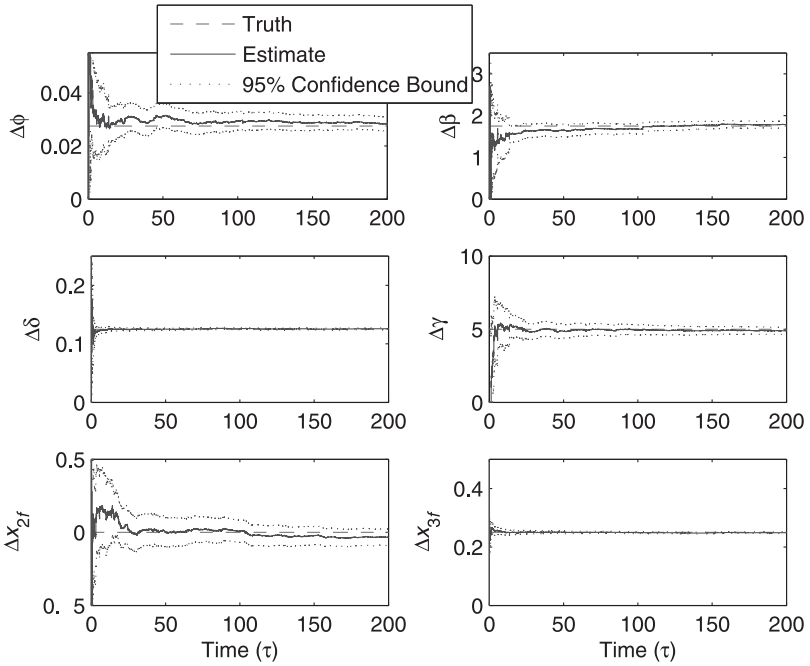


Fig. 2. Parameter estimation history using an EKF and data about four equilibrium points.

removed from the datasets, and the innovations are white. For many off-line applications, this approach works very well.

In summary, a stacked EKF is a fundamentally sound approach to parameter estimation; the stacked KF is usually not because the linearized system matrices typically yield biased results. When using the EKF, statistical confidence bounds should always be used in order to evaluate the accuracy of the estimates. These bounds are a function of the system observability, the number and size of the noises, and the amount of data used. Even with low observability, estimates created by running the EKF around a single equilibrium point can yield accurate results. Finally, the Kalman Smoother can result in improved estimation results over the EKF for off-line computations.

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Discussion on: "A Stacked Model Structure for Off-line Parameter Variation Estimation in Multi-equilibria Nonlinear Systems"

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In the paper by Collins and al. the problem of preventive diagnostics and maintenance of complex dynamic systems, deduced from the variations of the system parameters about their nominal values, has been studied. The authors have successfully applied the Extended Kalman Filter (EKF) in combination with the stacked nonlinear models to perform the parameters variations estimation.

The basic idea of the approach proposed by the authors can be summarized as a three step design:

—First transform the nonlinear dynamic system of the form

$$\dot{\chi}(t) = f(\chi(t), \mathcal{U}(t), P) + w(t), \quad \chi(0) = \chi_0 \tag{1}$$

$$Z(t) = h(\chi(t), \mathcal{U}(t), P) + v(t), \tag{2}$$

$$\dot{P} = 0, \tag{3}$$

in a linearized one given by

$$\begin{bmatrix} \dot{x}^{(l)}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A_c^{(l)} & B_{c,P}^{(l)} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x^{(l)}(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B_c^{(l)} \\ 0 \end{bmatrix} u^{(l)}(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} w(t),$$

$$\begin{bmatrix} x^{(l)}(0) \\ p(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta P \end{bmatrix}, \tag{4}$$

$$z^{(l)}(t) = \begin{bmatrix} C_c^{(l)} & D_{c,p}^{(l)} \end{bmatrix} \begin{bmatrix} x^{(l)}(t) \\ p(t) \end{bmatrix} + D_c^{(l)} u^{(l)}(t) + v(t) \tag{5}$$

—Then design a nonlinear filter when performing the estimation

—And finally estimate off line the parameter change

$$\Delta P \triangleq P - P_0$$

It is evident that the key of the problem solution lies in finding out the minimum number of equilibrium points needed and constructing an appropriate EKF to estimate the parameter variation. These represent the main contributions of this paper. However, the success of the filter in estimating the parameter change is the observability of ΔP in the data.

Estimation problem for more general classes on nonlinear plants will have to wait for further development of observer theory or be based on alternative concepts [1–3].

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